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THE EFFECT OF INITIAL DISPLACEMENT OF THE CENTER SUPPORT ON THE BUCKLING OF A COLUMN CONTINUOUS OVER THREE SUPPORTS

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THE EFFECT OF INITIAL DISPLACEMENT OF THE CENTER SUPPORT ON THE BUCKLING OF A COLUMN CONTINUOUS OVER THREE SUPPORTS

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SUMMARY

A long column continuous over three supports was tested to determine its critical load when the center support was given varying amounts of initial displacement. During each test the middle support was hinged so as to be free to move parallel to the column axis during buckling.

The critical loads predicted from load-deflection readings were different for the upper and lower spans. The larger predicted critical load in each test was for the span that, on buckling, deflected so as to deepen the initial deflection curve of the span and the smaller predicted critical load in each test was for the span that, on buckling, deflected so as to straighten out and reverse the initial deflection curve of the span. These observations hold regardless of whether the initial deflection of the center support was to the right or the left.

The difference between the critical loads predicted for the upper and lower spans is proportional to the initial deflection of the center support. The difference noted in these tests is not large in terms of errors permissible in practical design. The fact that a difference exists in the predicted critical loads suggests that an indiscriminate single application of the Southwell method as presented in reference 2, or as modified in reference 1, can result in definite and measurable errors.

The average of the predicted critical loads for the upper and lower spans is more correct than either predicted critical load. This observation suggests that whatever is causing the predicted critical load to be high in one span also causes the predicted critical load to be low in the other span.

The average of the predicted critical loads for the upper and lower spans is reduced by initial displacement of the center support and this reduction tends to increase with the absolute value of the initial displacement. In

those tests the reduction in the average critical load caused by initial displacement of the center support is very small. This fact indicates that the effect of curvature due to bending on the critical load for the compression flange material of a box beam is probably small and can be neglected in engineering design.

INTRODUCTION

In the course of a discussion with Lt. Col. Carl F. Greene, Air Corps Liaison Officer with the NACA, of the effect of curvature due to bending on the critical load for the compression flange material of a box beam, it was decided to test a long column continuous over three supports with the middle support given an initial displacement to represent the curvature of bending in a stressed-skin wing. In the test the middle support was hinged so as to be free to move parallel to the column axis during buckling. It was considered that this type of support would be a reasonable approximation to the type of support provided by the ribs of the box beam.

APPARATUS AND METHOD

The test set-up is shown in figures 1, 2, and 3. A diagrammatic sketch of the test is shown in figure 4.

The long continuous column used in the tests was a 3/4-inch-diameter steel bar 67-7/8 inches between the end knife edges. The middle of the continuous column was supported laterally by a stiff strut 12-7/8 inches long. One end of this strut was pin-joined to the continuous column at its middle. The axis of this pin joint was made to intersect the axis of the column so as to remove any possible adverse effects of an eccentric pin joint at this location. The other end of the lateral supporting strut was pin-joined to a rigid supporting structure in such manner that the middle of the continuous column could not deflect normal to the initial deflection.

During each test deflection readings at the middle of each span were taken from a fixed reference point on the slotted tension rod of the testing machine with an inside micrometer caliper reading to thousandths of an inch. The micrometer caliper and its extension bar are not shown in figures 1, 2, and 3.

In each test the specimen was loaded through the same range of loads. Therefore the small errors in the loads indicated by the testing machine cancel when comparing the results of one test with the results of another test.

RESULTS

The load-deflection readings taken during this investigation are given in tables I to VII inclusive. These data are plotted in figures 5 to 11 inclusive, from which the predicted loads are obtained in the manner of reference 1. These predicted loads are listed in table VIII. In figure 12 the difference between the predicted critical load for the upper and lower spans is plotted against the initial deflection of the center support. In figure 13 the average value of the predicted critical loads is plotted against the initial deflection of the center support.

In each test buckling occurred with deflection to the right in the upper span and deflection to the left in the lower span. The test for which the initial deflection of the center support was 0.749 inch was the last test performed. In this test the column was loaded to destruction and the maximum load was found to be 3810 pounds.

CONCLUDING DISCUSSION

Inspection of table VIII shows that the critical loads predicted from load-deflection readings were different for the upper and lower spans. The larger predicted critical load in each test was for the span that, on buckling, deflected so as to deepen the initial deflection curve of the span and the smaller predicted critical load in each test was for the span that, on buckling, deflected so as to straighten out and reverse the initial deflection curve of the span. These observations hold regardless of whether the initial deflection of the center support was to the right or the left.

Figure 12 shows that the difference between the critical load predicted for the upper and lower spans is proportional to the initial deflection of the center support. The difference noted in these tests is not large in terms of errors permissible in practical design. The fact that a difference exists in the predicted critical loads suggests that an indiscriminate single application of the Southwell method, as presented in reference 2 or as modified in reference 1, can result in definite and measurable errors. It is therefore desirable to study the cause of the difference in the predicted critical loads in order to determine whether or not the error could ever become large enough to be of practical importance in engineering applications.

In the one test that was carried to destruction, the following values were obtained:

Predicted critical load, upper span	3897 lb
Predicted critical load, lower span	3757 lb
Average predicted critical load	3827 lb
Maximum load in destruction test	3810 lb

From these results it is concluded that the average of the predicted critical loads for the upper and lower spans is more correct than either predicted critical load. This observation suggests that whatever is causing the predicted critical load to be high in one span also causes the predicted critical load to be low in the other span.

It is concluded from figure 13 that the average value of the predicted critical loads is reduced by initial displacement of the center support and this reduction tends to increase with the absolute value of the initial displacement. In these tests the reduction in the average predicted critical load caused by initial displacement of the center support is, however, very small. This fact indicates that the effect of curvature due to bending on the critical load for the compression flange material of a box beam is probably small and can be neglected in engineering design.

The fact that negative initial displacement of the center support gave lower average predicted critical loads than corresponding positive initial displacements indicates that there may have been a lack of perfect symmetry and central loading. The fact that, on buckling, the upper span always deflected to the right and the lower span to the left seems to support the suggestion that perfect symmetry and central loading were not achieved.

It is possible that a difference in the loading conditions in the two spans when the center support is initially deflected causes the predicted critical loads for the two spans to differ. Certainly a difference in loading exists when deflection, on buckling, deepens the initial deflection curve of one span and straightens out and reverses the initial deflection curve of the other span. Inspection of tables II to VII inclusive shows that for the same increment of load $P-P_1$, the larger increment of deflection $y-y_1$ is always obtained when the deflection, on buckling, deepens the initial deflection curve of the span. The existence of different increments of deflection for the same increment of load can only mean a difference in the loading conditions for the two spans.

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REFERENCES

1. Lundquist, Eugene E.: Generalized Analysis of Experimental Observations in Problems of Elastic Stability. T.N. No. 658, NACA, 1938.

2. Southwell, R. V.: On the Analysis of Experimental Observations in Problems of Elastic Stability. Proc., Royal Soc., A, vol. 135, 1932, pp. 601-616.

TABLE I

Load-Deflection Data

Initial Deflection at Center Support 0 inches.

P (lb)	Upper Span P_1 = 3000 lb. y_1 = 18.191 in.				Lower Span P_1 = 3000 lb. y_1 = 18.114 in.			
	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)
3000	18.191	0	0		18.114	0	0	
3200	18.204	.013	200	0.0000650	18.104	-.010	200	-0.0000500
3400	18.224	.033	400	.0000825	18.083	-.031	400	- .0000775
3500	18.244	.053	500	.0001060	18.064	-.050	500	- .0001000
3600	18.275	.084	600	.0001400	18.031	-.083	600	- .0001383
3700	18.350	.159	700	.0002271	17.954	-.160	700	- .0002286
3750	18.451	.260	750	.0003467	17.854	-.260	750	- .0003467

TABLE II

Load-Deflection Data

Initial Deflection at Center Support 0.453 inches.

P (lb)	Upper Span P_1 = 3000 lb. y_1 = 18.556 in.				Lower Span P_1 = 3000 lb. y_1 = 18.470 in.			
	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)
3000	18.556	0	0		18.470	0	0	
3200	18.576	.020	200	0.0001000	18.461	-.009	200	-0.0000450
3400	18.605	.049	400	.0001225	18.444	-.026	400	-.0000650
3500	18.635	.079	500	.0001580	18.424	-.046	500	-.0000920
3600	18.677	.121	600	.0002017	18.385	-.085	600	-.0001417
3700	18.796	.240	700	.0003429	18.269	-.201	700	-.0002871

TABLE III

Load-Deflection Data

Initial Deflection at Center Support -0.447 inches.

P (lb)	Upper Span P_1 = 3000 lb. y_1 = 17.833 in.				Lower Span P_1 = 3000 lb. y_1 = 17.741 in.			
	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)
3000	17.833	0	0		17.741	0	0	
3200	17.843	.010	200	0.0000500	17.724	-.017	200	-0.0000850
3400	17.863	.030	400	.0000750	17.691	-.050	400	-.0001250
3500	17.885	.052	500	.0001040	17.660	-.081	500	-.0001620
3600	17.933	.100	600	.0001667	17.607	-.134	600	-.0002233
3650	17.985	.152	650	.0002338	17.547	-.194	650	-.0002985

TABLE IV

Load-Deflection Data

Initial Deflection at Center Support 0.749 inches.

P (lb)	Upper Span P_1 = 3000 lb. y_1 = 18.742 in.				Lower Span P_1 = 3000 lb. y_1 = 18.662 in.			
	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)
3000	18.742	0	0		18.662	0	0	
3200	18.757	.015	200	0.0000750	18.663	+.001	200	+0.0000050
3400	18.782	.040	400	.0001000	18.656	-.006	400	-.0000150
3500	18.811	.069	500	.0001380	18.647	-.015	500	-.0000300
3700	18.901	.159	700	.0002271	18.575	-.087	700	-.0001243

TABLE V

Load-Deflection Data

Initial Deflection at Center Support -0.747 inches.

P (lb)	Upper Span P_1 = 3000 lb. y_1 = 17.605 in.				Lower Span P_1 = 3000 lb. y_1 = 17.490 in.			
	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)
3000	17.605	0	0		17.490	0	0	
3200	17.614	.009	200	0.0000450	17.463	-.027	200	-0.0001350
3400	17.635	.030	400	.0000750	17.422	-.068	400	-.0001700
3500	17.662	.057	500	.0001140	17.381	-.109	500	-.0002180
3600	17.719	.114	600	.0001900	17.309	-.181	600	-.0003017
3650	17.818	.213	650	.0003277	17.226	-.264	650	-.0004062

TABLE VI

Load-Deflection Data

Initial Deflection at Center Support 1.013 inches.

P (lb)	Upper Span P_1 = 3000 lb. y_1 = 19.010 in.				Lower Span P_1 = 3000 lb. y_1 = 18.902 in.			
	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)
3000	19.010	0	0		18.902	0	0	
3200	19.037	.027	200	0.0001350	18.897	-.005	200	-0.0000250
3400	19.080	.070	400	.0001750	18.880	-.022	400	-.0000550
3500	19.111	.101	500	.0002020	18.856	-.046	500	-.0000920
3600	19.169	.159	600	.0002650	18.804	-.098	600	-.0001633
3650	19.254	.244	650	.0003754	18.739	-.163	650	-.0002508

TABLE VII

Load-Deflection Data

Initial Deflection at Center Support -1.028 inches.

P	Upper Span P_1 = 3000 lb. y_1 = 17.393 in.				Lower Span P_1 = 3000 lb. y_1 = 17.274 in.			
(lb)	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)	y (in.)	$y-y_1$ (in.)	$P-P_1$ (lb)	$\frac{y-y_1}{P-P_1}$ (in/lb)
3000	17.393	0	0		17.274	0	0	
3200	17.396	.003	200	0.0000150	17.241	-.033	200	-0.0001650
3400	17.416	.023	400	.0000575	17.193	-.081	400	-.0002025
3500	17.450	.057	500	.0001140	17.150	-.124	500	-.0002480
3600	17.499	.106	600	.0001767	17.075	-.199	600	-.0003317
3650	17.566	.173	650	.0002662	16.996	-.278	650	-.0004277

TABLE VIII

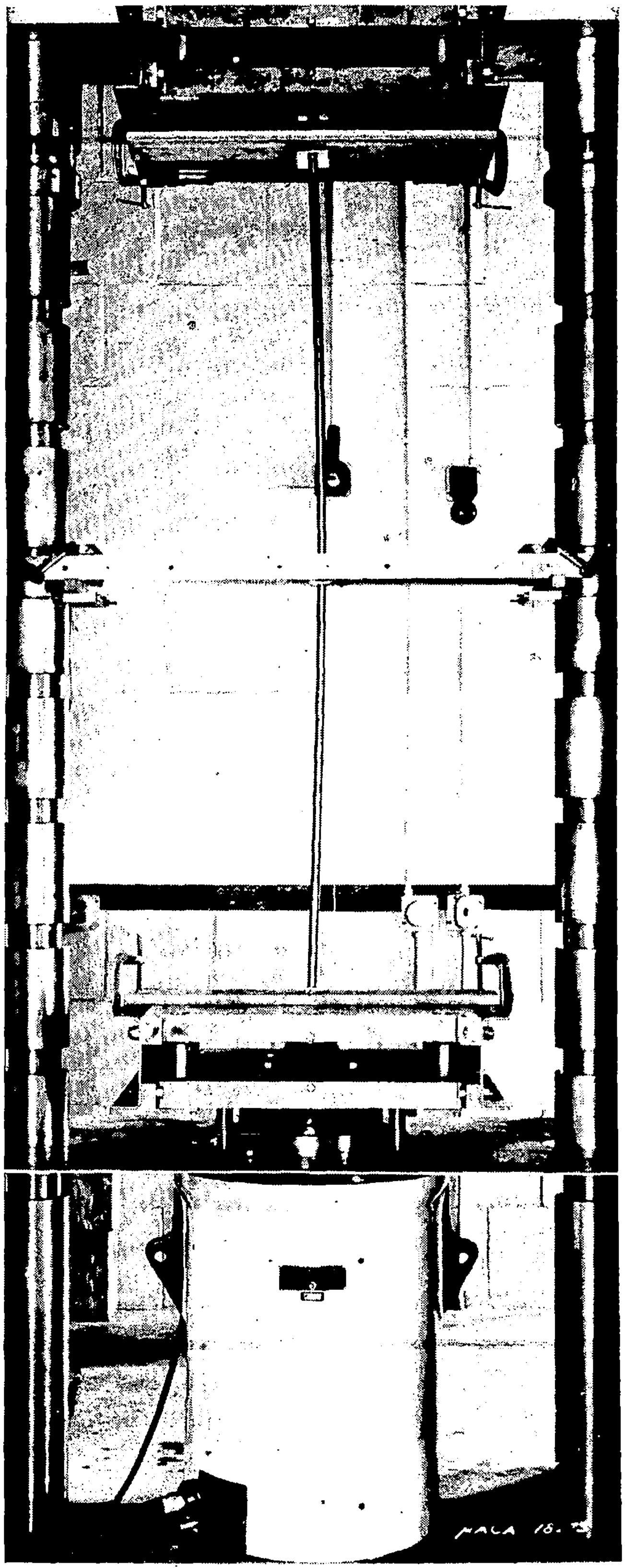
Summary of Critical Loads

Predicted From Load-Deflection Data.

Initial Deflection at Center Support (in.)	P_{cr} Upper Span (lb)	P_{cr} Lower Span (lb)	P_{cr} Average (lb)	P_{cr} Upper Span − P_{cr} Lower Span (lb)
0	3858	3850	3854	8
.453	3868*	3789**	3829	79
-.447	3773**	3830*	3802	-57
.749	3897*	3757**	3827	140
-.747	3727**	3831*	3779	-104
1.013	3890*	3733**	3812	157
-1.020	3737**	3872*	3805	-135

*Deflected on buckling so as to deepen the initial deflection curve of the span.

**Deflected on buckling so as to straighten out and reverse the initial deflection curve of the span.



300,000-POUND
HYDRAULIC
COMPRESSION
TESTING
MACHINE

Figure 1.- Test set-up showing column with initial deflection at center support of 0.749 inch before loading.

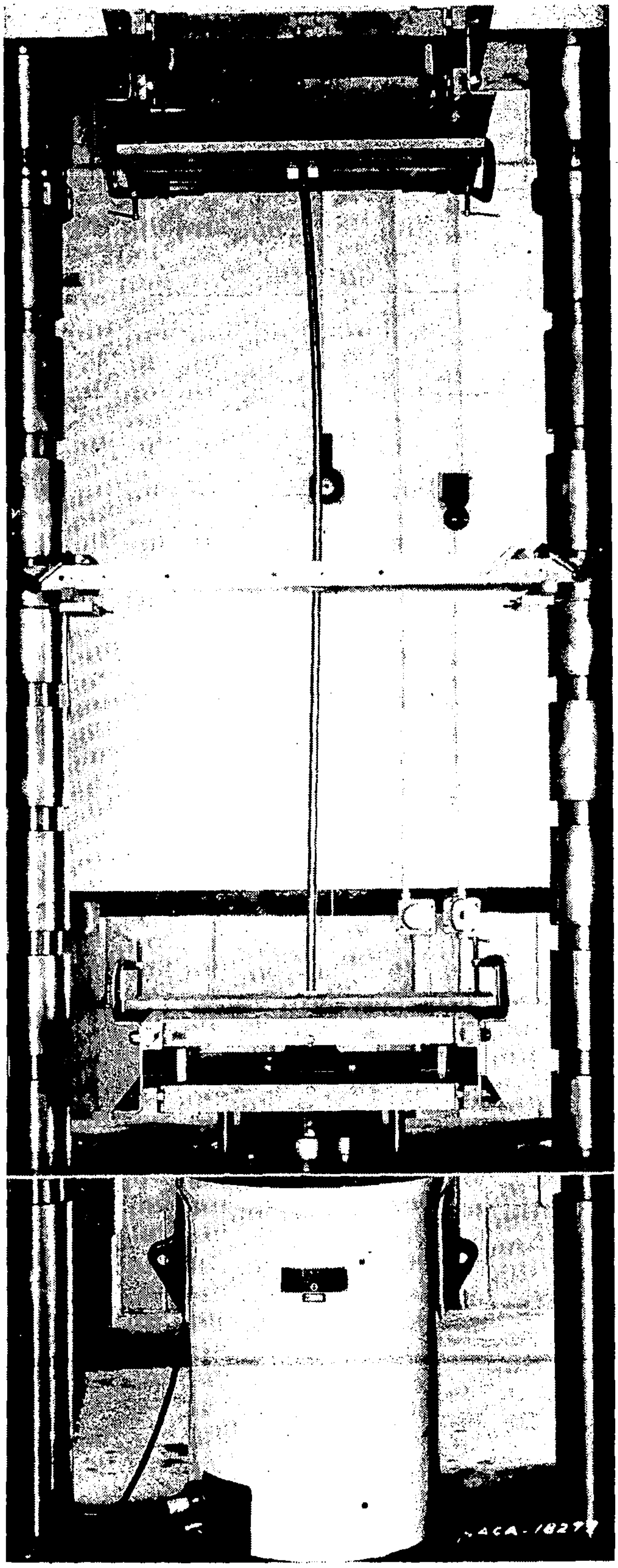
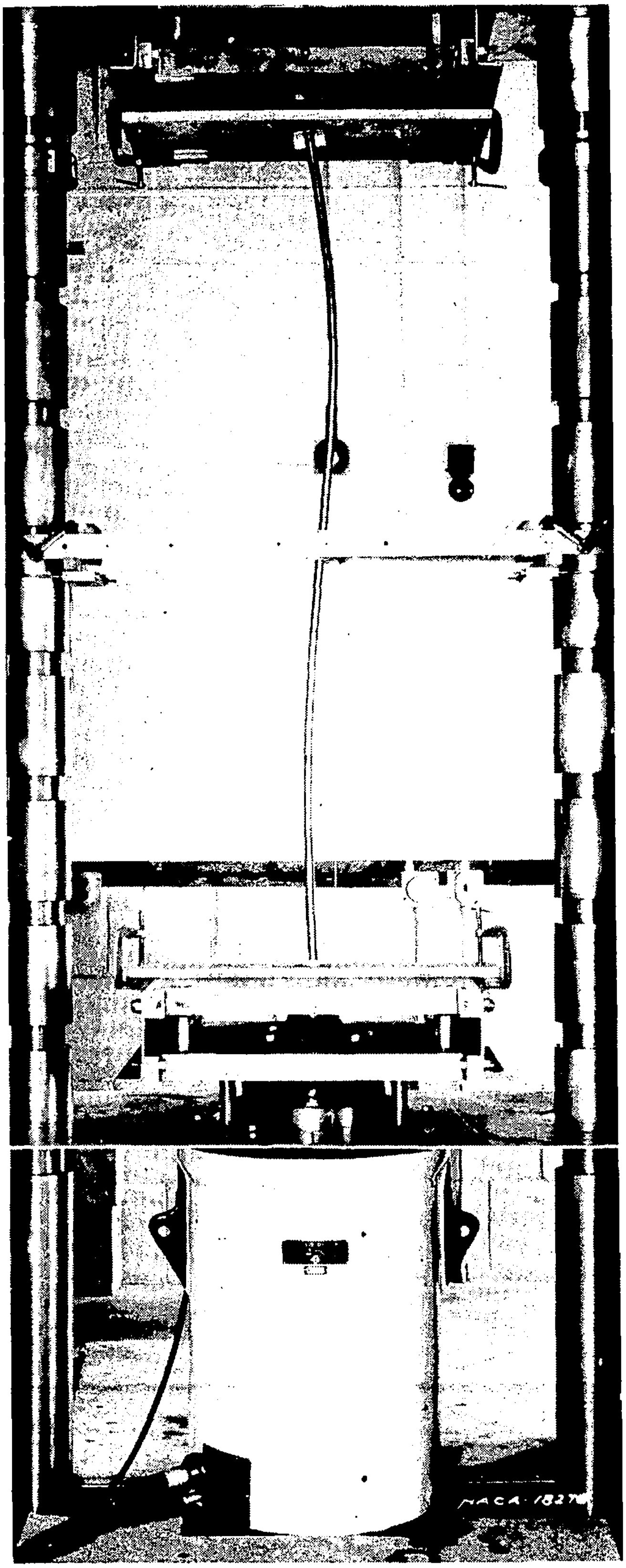


Figure 2.- Test set-up showing column with initial deflection at center support of 0.749 inch approaching critical load.



300,000-POUND HYDRAULIC COMPRESSION TESTING MACHINE

Figure 3.- Test set-up showing column with initial deflection at center support of 0.749 inch at, or past, maximum load.

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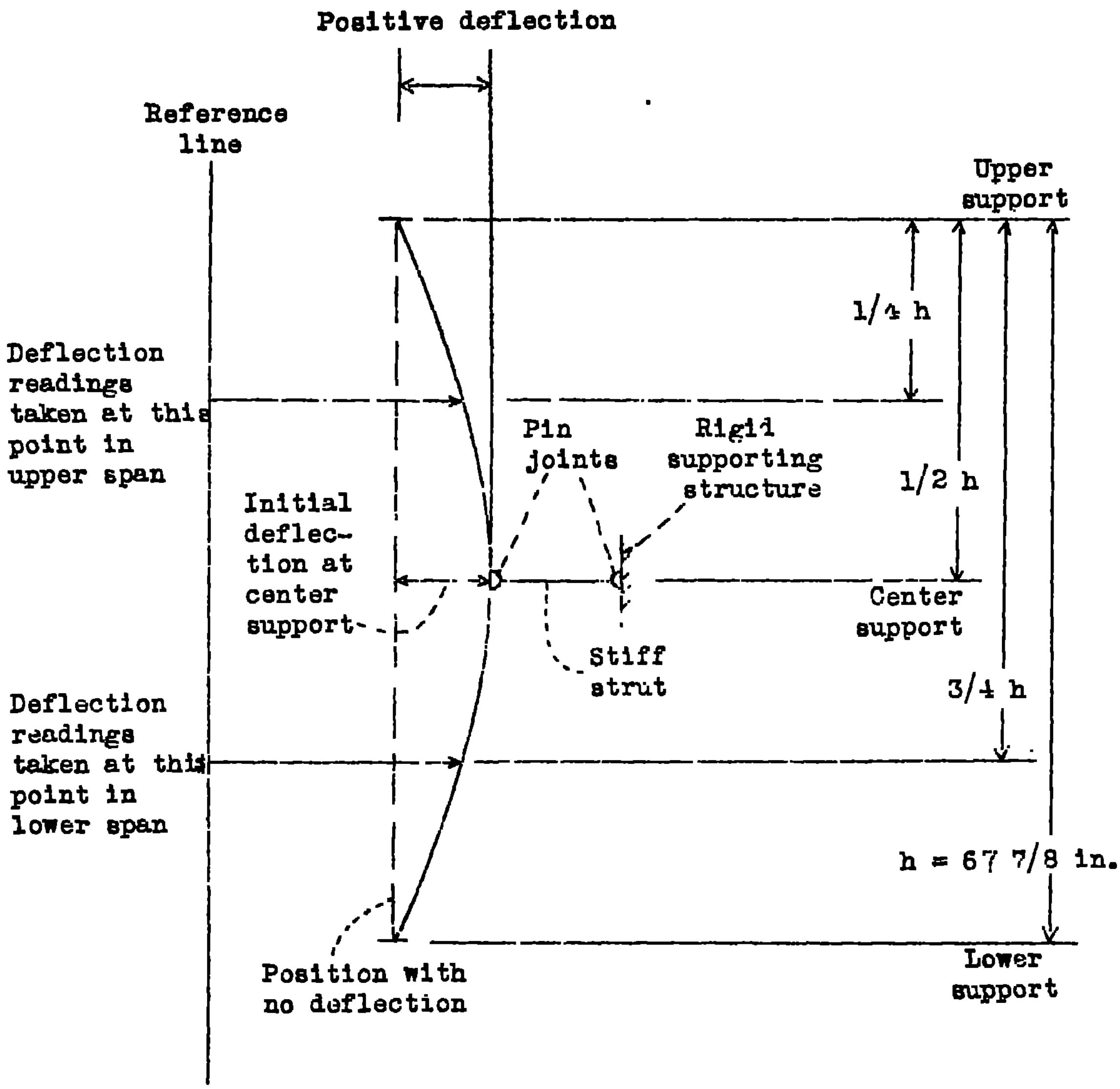

Figure 4.- Diagrammatic sketch of test specimen.

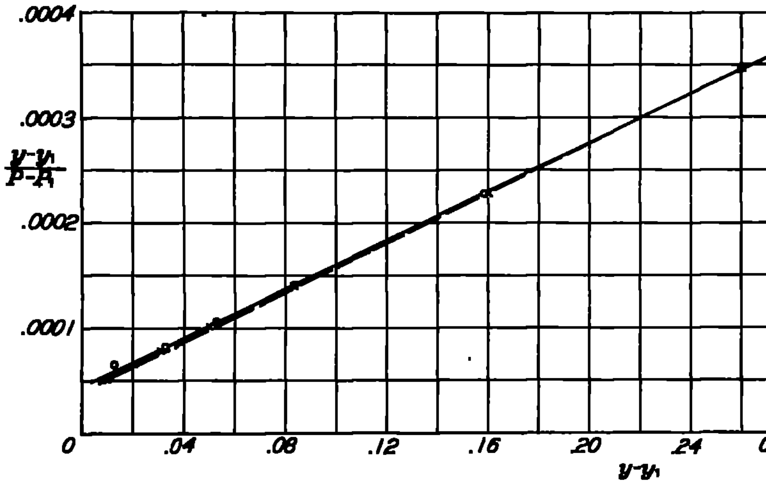

Figure 5.-

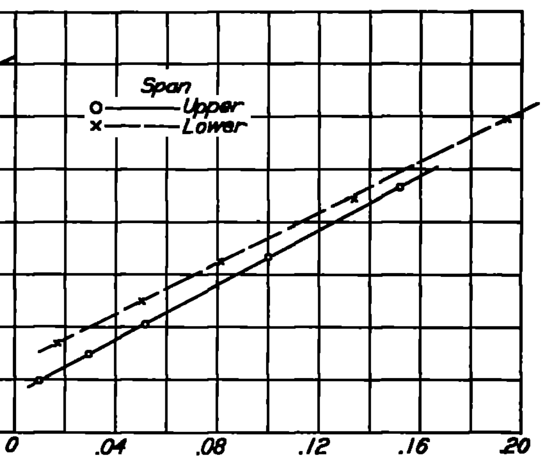

Figure 7.-

Figures 5,7.- Graphs of load-deflection data: Fig.5, no initial deflection at center support; Fig.7, initial deflection at center support, -0.447 inch.

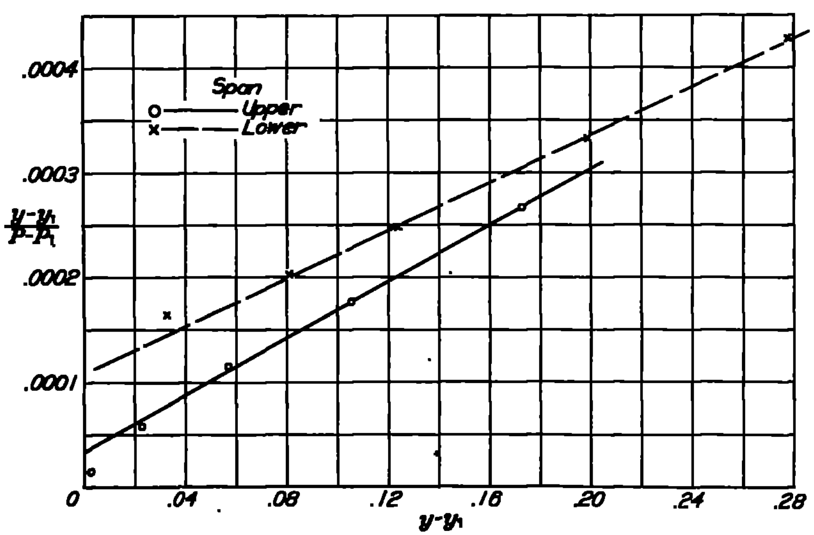

Figure 11.- Graph of load-deflection data. Initial deflection at center support, -1.020 inches.

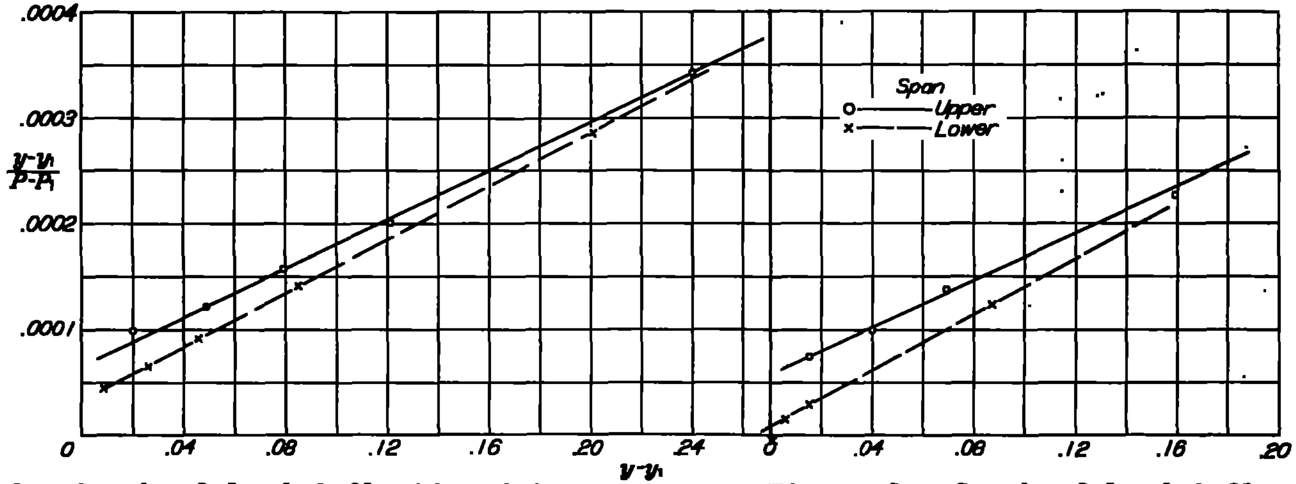

Figure 6.- Graph of load-deflection data. Initial deflection at center support, 0.453 inch.

Figure 8.- Graph of load-deflection data. Initial deflection at center support, 0.749 inch.

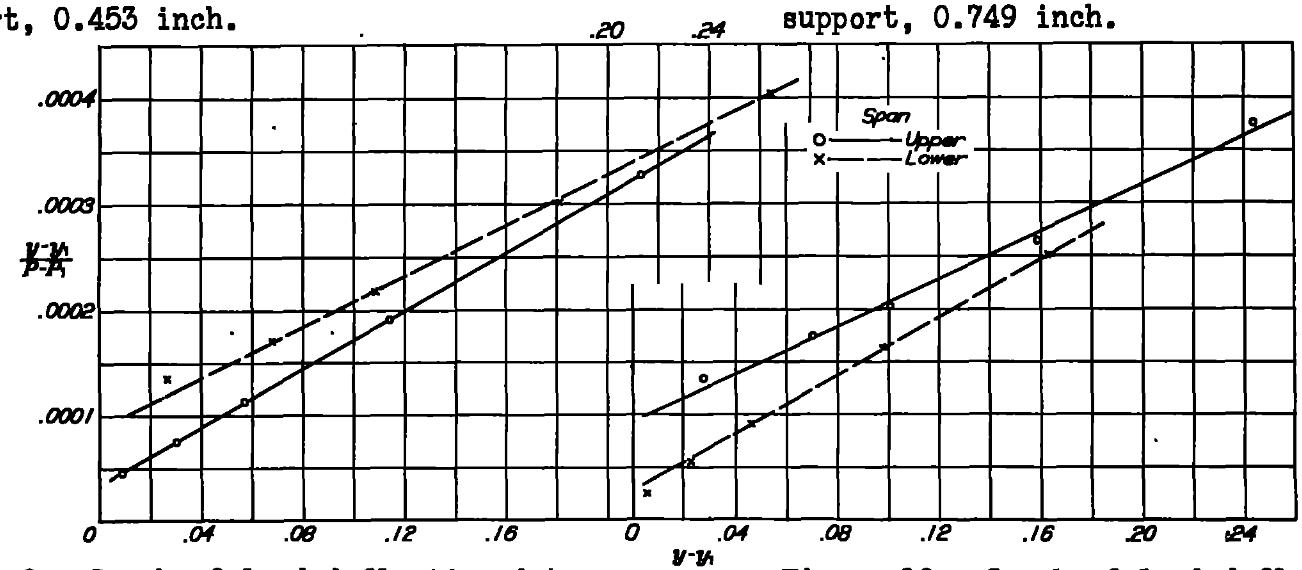

Figure 9.- Graph of load-deflection data. Initial deflection at center support, -0.747 inch.

Figure 10.- Graph of load-deflection data. Initial deflection at center support, 1.013 inches.

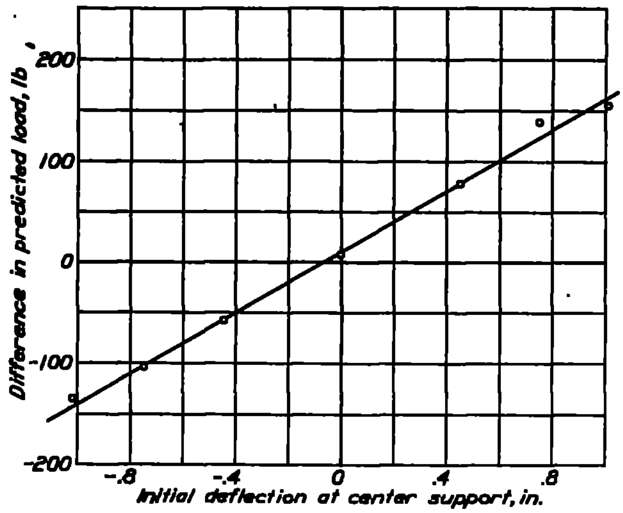

Figure 12.- Variation of difference in predicted critical load with initial deflection of the center support.

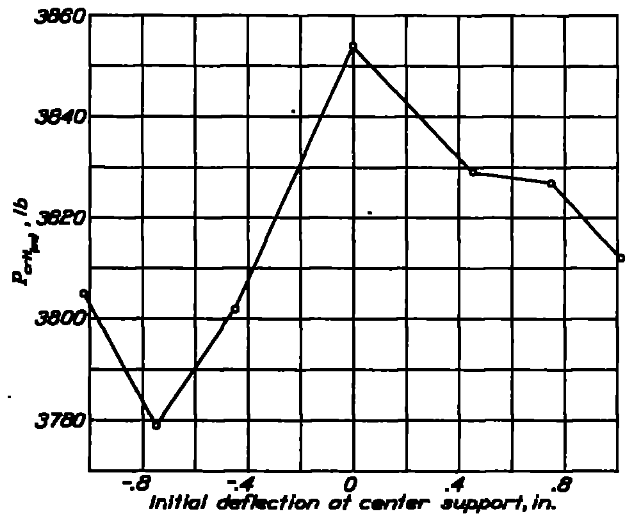

Figure 13.- Variation of average predicted critical load with initial deflection of center support.